

Data driven approach for stochastic data envelopment analysis

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ABSTRACT

Decision making based on data driven deals with a large amount of data will evaluate the process's effectiveness. Evaluate effectiveness in this paper is measure of performance efficiency of data envelopment analysis (DEA) method in this study is the approach with uncertainty problems. This study proposed a new method called the robust stochastic DEA (RSDEA) to approach performance efficiency in tackling uncertainty problems (i.e., stochastic and robust optimization). The RSDEA method develops to combine the stochastic DEA (SDEA) formulation method and Robust Optimization. The numerical example demonstrates the performance efficiency of the proposed formulation method, with the results performing confirmed that the efficiency value is 89%.

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1. INTRODUCTION

Today, the development of information technology in data driven decision making is an important thing in business world. The idea of the data driven decision making is that every decision based on data must be generalized from an effective set of primary information. Decision making is not only related with data but also related to measure of performance efficiency. Performance efficiency measurement deals with data envelopment analysis (DEA). In classic concept, DEA is non-parametric technique first proposed by [1] called Charnes, Cooper and Rhodes (CCR) model. This model using linear programming technique that produce a single efficiency measurement. Base of DEA is a linear programming technique to evaluate the relative efficiency of multiinput and multi multioutput decision making unit (DMU) on the observed data [2] by comparing DMU one with others that the utilities is same resources to produce the same output [3].

The main goal of DEA is intended as a method for performance evaluation and benchmarking [4]. DEA techniques have been widely used by many researchers for various field, including: measurement of efficiency radiotherapy treatment [5]; hotel performance evaluation [6]; evaluation of efficiency of logistic sustainability performance [7]; evaluation of teaching performance in university [8]; and efficiency analysis in emergency departments [9], [10]. The efficiency evaluation process often involves a stochastic approach because of uncertainty inherent in the data in real life problems [11], [12], the stochastic DEA model using input and output data with stochastic variations, and discussed develop an algorithm to help organization for evaluating performance in stochastic problems. Khodabakhshi [13], describes the superefficiency problem based on the input relaxation model in the DEA stochastic model. The relaxation superefficiency input model was developed through stochastic DEA (SDEA), deterministic equivalence, and also nonlinear program derivatives. Khodabakhshi and Asgharian [14], describes an input relaxation model and a stochastic version

into DEA that uses more flexibility of change that using combination of inputs to find the maximum possible output. The optimization models, such as stochastic programming, chance constraint programming, and robust optimization (RO) have been proposed to support decision making in uncertainty problems [15].

Motivation this study is how to determinate of evaluation performance based on data with DEA framework with combine SDEA model and robust optimization model. The SDEA formulation methods in this studies proposed by [16]. Determination efficiency problems with DEA contains of large data is difficult. This related to the completion of liner programming with large data. Therefore, this level of difficulty increase with uncertainty input and output [17]. Hence, the authors proposed the methods robust stochastic data envelopment analysis as an approaches model to evalatuate performance efficiency in uncertainty problems. To emphasise performance of efficiencies the proposed model, a comparison with SDEA model is made.

This study is organized into sections as: section 2 presents methods and materials, explains the based DEA method, stochastic DEA (SDEA), robust optimization, and develops formulation robust stochastic DEA (RSDEA) and a numerical example. Section 3 explains the result and discussion. Section 4 refers to the conclusion about studies.

2. METHOD AND MATERIALS

This section will introduce the concept of the based DEA model as a base of the framework in this research and preparation structured to build SDEA and the new RSDEA method.

2.1. Based method of DEA

Suppose there are n DMUs, each of them have varying amounts of m different inputs to produce s different outputs denoted by x_{ij}, \dots, x_{mj} and y_{ij}, \dots, y_{sj} , as the respective input and output vectors of DMU_j . This study using the based method of DEA provided by [1] as:

$$\maximize_{u,y} \sum_{r=1}^s u_r y_{r0} \quad (1)$$

Subject to:

$$\begin{aligned} \sum_{i=1}^m v_i x_{i0} &= 1 \\ \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} &\leq 0 \\ j &= 1, \dots, n \\ v_i &\geq 0 \quad i = 1, \dots, m \\ u_r &\geq 0 \quad r = 1, \dots, s \end{aligned}$$

2.2. SDEA

The following development SDEA methods assume a stochastic framework with all input variables transformed to output variables. Let variable of $DMU_s (s = 1, 2, 3, \dots, r)$ which all of them have random behavior. Let us further assume that $\tilde{\mathbf{u}}_r = (\tilde{u}_{1r}, \tilde{u}_{2r}, \dots, \tilde{u}_{sr})$ for each $\tilde{u}_{rj} (r = 1, \dots, s)$ denoting output components has probability distribution. By maximizing the expected of efficiency evaluated DMU, as:

$$\max E(y^t \tilde{u}_0) \quad (2)$$

Subject to:

$$\begin{aligned} \text{prob}(y^t \tilde{u}_j \leq \beta_j) &\geq 1 \\ u &\geq 0 \end{aligned}$$

In (2) transformed into a deterministic DEA form by considered from stochastic DEA. In [18], model Chance constraint programming have new variables with standard normal distribution, as following model:

$$\tilde{z}_j = \frac{y^t(\tilde{u}_j - \bar{u}_j)}{\sqrt{\text{var}_j}}, j = 1, 2, \dots, N \quad (3)$$

By substituted (2), (3) as:

$$\text{prob} \left(\tilde{z}_j \leq \frac{\beta_j - y^t \bar{u}_j}{\sqrt{\text{var}_j}} \right) \geq 1 - \alpha_j \quad (4)$$

Furthermore, with simply converted into following form:

$$\frac{\beta_j - y^t \bar{u}_j}{\sqrt{\text{var}_j}} \geq \phi^{-1}(1 - \alpha_j), j = 1, 2, \dots, N \quad (5)$$

From (2), (5), simply transformation as following equivalent linear programming model as follows:

$$\text{Maximize } y^t \bar{u}_0 \quad (6)$$

Subject to:

$$\beta_j - y^t \bar{u}_j \geq y^t b_j \phi^{-1}(1 - \alpha_j), j = 1, 2, \dots, N \quad (7)$$

2.3. Robust optimization

Robust optimization model when decision makers have to choose a strategy without knowing the exact value taken from an uncertain parameter. Robust optimization become popular approach as a robust methodology for solving the model optimization in uncertainty problems by providing a feasible solution in it's the objective function. This approach to optimized mathematical uncertainty and maintain tractability of the model as proposed by [19]. Robust Optimization Model has been used to make decisions in dynamic environments where future decisions depends on the realization of data such as inventory issues [20], contracts [21] and project expansion planning [22]. Uncertainty has two forms: (i) estimation for constant value parameter, (ii) stochastic random variables. Optimization developed based on 2 principles [23]–[25], a modern practice in operations uncertainty management: i) estimating with often fail point and must be replaced with range estimates; ii) estimating with aggregate are more accurate than individual estimates. To present the robust framework in mathematics way [26] consider following linier programming problem:

Minimize $c'x$

Subject to: $Ax \geq b$,

$$x \in X \quad (8)$$

One of based issues of (6), the uncertainty problem about feasebility. In particular, the decision maker will ensure that every constraint is included for the possible value of A within a certain specified uncertainty of A . The set A is assumed each a_{ij} coefficient of matrix A is an uncertainty constraint, and all coefficients are independent. The decision maker knows the various estimates for all the uncertain parameters, in particular the parameter a_{ij} belongs to a symmetrical interval $[\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]$ centered on the approximate point a_{ij} when measuring the accuracy of estimate.

2.4. Developed RSDEA method

In this study, the formulation of the proposed RSDEA aims to maximize the efficiency of the multi-uncertainty output of the stochastic DEA (SDEA) methods and its robust optimization. The formulation method is an equivalent deterministic model to be solved using linear programming.

Let for each variable of $DMU_s (s = 1, 2, 3, \dots, r)$ has random with expressed by $\tilde{u}_r = (\tilde{u}_{1r}, \tilde{u}_{2r}, \dots, \tilde{u}_{sr})$, for each $\tilde{u}_{tr} (t = 1, 2, 3, \dots, s)$ also suppose all in variable are jointly probability distributed. The uncertainty of DEA for (\tilde{x}_{ij}) input or (\tilde{y}_{ij}) output are data wich is developed by the error. By the maximize the objective of (6), then the Robust DEA method be given as:

Maximize $c'x$
 Subject to:

$$\begin{aligned}
 \sum_{j=1}^n c'x &\leq \tilde{b}_i \\
 \sum_{i=1}^m \tilde{v}_i \tilde{x}_{i0} &= 1 \\
 \sum_{r=1}^s \tilde{u}_r \tilde{y}_{rj} - \sum_{i=1}^m \tilde{v}_i \tilde{x}_{ij} &\leq 0; j = 1, \dots, n \\
 \tilde{v}_i &\geq 0; i = 1, \dots, m \\
 \tilde{u}_r &\geq 0; r = 1, \dots, s
 \end{aligned} \tag{8}$$

Substitute (8) with (2) as:

$$\text{Maximize: } \sum_{r=1}^s \tilde{u}_r \tilde{y}_{r0} \tag{9}$$

Subject to: $\sum_{r=1}^s \tilde{u}_r \tilde{y}_{r0} \leq b_i$

$$\begin{aligned}
 \sum_{i=1}^m \tilde{v}_i \tilde{x}_{i0} &= 1 \\
 \sum_{r=1}^s \tilde{u}_r \tilde{y}_{rj} - \sum_{i=1}^m \tilde{v}_i \tilde{x}_{ij} &\leq 0 \\
 \tilde{v}_i &\geq 0
 \end{aligned}$$

Since (9), at the first constraint $\sum_{r=1}^s \tilde{u}_r \tilde{y}_{r0} \leq b_i$, reformulated to giving the score of super efficiency. In (9) will change with the constraint at (7) in order to the constraint not deviation on surplus variable M . Further at the constraint (7) there is an inequality greater than equal to, so the constraint (7) reformulated as:

$$y^t(b_j \phi^{-1}(1 - \alpha_j) + \tilde{u}_j) \leq \beta_j \quad j = 1, \dots, N \tag{10}$$

Let $b_j = 1$ and all of input convert to be output with α = risk level and also β = aspiration level. Then the formulation (9) and (10) called RSDEA can be formulated as:

$$\text{Maximize } \sum_{r=1}^s \tilde{u}_r \tilde{y}_{r0} \tag{11}$$

$$\text{Subject to : } y^t(b_j \phi^{-1}(1 - \alpha_j) + \tilde{u}_j) \leq \beta_j \quad j = 1, \dots, N \tag{12}$$

$$\sum_{i=1}^m \tilde{v}_i \tilde{x}_{i0} = 1 \tag{13}$$

$$\sum_{r=1}^s \tilde{u}_r \tilde{y}_{rj} - \sum_{i=1}^m \tilde{v}_i \tilde{x}_{ij} \leq 0 \quad j = 1, \dots, n \tag{14}$$

$$(\tilde{v}_i, \tilde{u}_r) \geq 0 \quad i = 1, \dots, m; r = 1, \dots, s$$

In other words, the formulation formed above produces a method formulation with the objective function of maximizing the expectation of DMU evaluation on the number of outputs by considering the following constraints: a. multiplying the cumulative distribution function of the output is less than or equal to the value of the aspiration level, b. the result of adding up all the inputs must be equal one, c. the result of the difference between the number of outputs and inputs must be less than zero.

2.4. Numerical example

To understand the performance of the proposed RSDEA, we implement proposed (11) to (14) to analyze the efficiency performance of the college. For estimating the efficiency performance measure with this RSDEA, this study adopts 12 DMUs with four inputs and four outputs. In this case, given value of $\alpha = 0.2$, $\beta = 0.9$ and $\phi^{-1} = 1.0625$. Table 1 gives the information data of inputs and outputs. All of the data inputs are the mean of the number of lecturers, students, research of funding, employees. As well as the outputs data which is the mean of the number of alumni, research publications, and LPM.

Table 1. Information data of DMUs with 4 inputs and outputs

DMU	I ₁	I ₂	I ₃	I ₄	O ₁	O ₂	O ₃	O ₄
DMU ₁	17	588	20	5	610	5	5	5
DMU ₂	26	747	25	6	533	5	5	5
DMU ₃	15	396	15	4	195	5	4	5
DMU ₄	17	467	20	3	300	5	5	5
DMU ₅	25	348	25	2	252	5	7	5
DMU ₆	23	499	20	2	224	5	5	5
DMU ₇	19	420	22	1	326	5	6	5
DMU ₈	17	689	20	4	273	5	5	5
DMU ₉	34	822	35	5	284	5	4	5
DMU ₁₀	10	501	10	7	204	5	4	5
DMU ₁₁	11	719	10	4	273	5	8	5
DMU ₁₂	11	262	10	2	183	5	4	5

3. RESULTS AND DISCUSSION

This study aims to develop the new robust stochastic DEA (RSDEA) methods to tackle uncertainty problems (i.e., stochastic and robust optimization. The objective function of the RSDEA is to maximize the efficiency of multi-output from the uncertain stochastic model and robust optimization. In this case, input variable denotes v and output variable denotes u . The numerical example as performed the proposed methods to determine performance efficiency, we use the case study to maximize the output to get efficiency performance of the college. On the Table 1 show 12 DMUs with 4 inputs and 4 outputs. Furthermore, based on a case, we solved the formulation 12 times according to the number of DMUs, with each formulation having the same constraints. Developing this formula was extended from the formulation of the proposed method for DMU_1 as according to this study case as:

$$\text{Maximize } 610u_1 + 5u_2 + 5u_3 + 5u_4 \quad (15)$$

Subject to:

$$610.85u_1 + 5.85u_2 + 5.85u_3 + 5.85u_4 + 17.85u_5 + 588.85u_6 + 200.85u_7 + 5.85u_8 \leq 0.9 \quad (16)$$

$$533.85u_1 + 5.85u_2 + 5.85u_3 + 5.85u_4 + 26.85u_5 + 747.85u_6 + 250.85u_7 + 6.85u_8 \leq 0.9 \quad (17)$$

$$195.85u_1 + 5.85u_2 + 4.85u_3 + 5.85u_4 + 15.85u_5 + 396.85u_6 + 150.85u_7 + 4.85u_8 \leq 0.9 \quad (18)$$

$$300.85u_1 + 5.85u_2 + 5.85u_3 + 5.85u_4 + 17.85u_5 + 467.85u_6 + 200.85u_7 + 3.85u_8 \leq 0.9 \quad (19)$$

$$252.85u_1 + 5.85u_2 + 7.85u_3 + 5.85u_4 + 25.85u_5 + 348.85u_6 + 250.85u_7 + 2.85u_8 \leq 0.9 \quad (20)$$

$$224.85u_1 + 5.85u_2 + 5.85u_3 + 5.85u_4 + 23.85u_5 + 499.85u_6 + 200.85u_7 + 2.85u_8 \leq 0.9 \quad (21)$$

$$326.85u_1 + 5.85u_2 + 6.85u_3 + 5.85u_4 + 19.85u_5 + 420.85u_6 + 220.85u_7 + 1.85u_8 \leq 0.9 \quad (22)$$

$$273.85u_1 + 5.85u_2 + 5.85u_3 + 5.85u_4 + 17.85u_5 + 689.85u_6 + 200.85u_7 + 4.85u_8 \leq 0.9 \quad (23)$$

$$284.85u_1 + 5.85u_2 + 4.85u_3 + 5.85u_4 + 34.85u_5 + 822.85u_6 + 350.85u_7 + 5.85u_8 \leq 0.9 \quad (24)$$

$$204.85u_1 + 5.85u_2 + 4.85u_3 + 5.85u_4 + 10.85u_5 + 501.85u_6 + 100.85u_7 + 7.85u_8 \leq 0.9 \quad (25)$$

$$273.85u_1 + 5.85u_2 + 8.85u_3 + 5.85u_4 + 11.85u_5 + 719.85u_6 + 100.85u_7 + 4.85u_8 \leq 0.9 \quad (26)$$

$$183.85u_1 + 5.85u_2 + 4.85u_3 + 5.85u_4 + 11.85u_5 + 262.85u_6 + 100.85u_7 + 2.85u_8 \leq 0.9 \quad (27)$$

$$17v_1 + 588v_2 + 200v_3 + 5v_4 = 1 \quad (28)$$

$$610u_1 + 5u_2 + 5u_3 + 5u_4 - 17v_1 - 588v_2 - 200v_3 - 5v_4 \leq 0 \quad (29)$$

$$533u_1 + 5u_2 + 5u_3 + 5u_4 - 26v_1 - 747v_2 - 250v_3 - 6v_4 \leq 0 \quad (30)$$

$$195u_1 + 5u_2 + 4u_3 + 5u_4 - 150v_1 - 396v_2 - 150v_3 - 4v_4 \leq 0 \quad (31)$$

$$300u_1 + 5u_2 + 5u_3 + 5u_4 - 17v_1 - 467v_2 - 200v_3 - 3v_4 \leq 0 \quad (32)$$

$$252u_1 + 5u_2 + 7u_3 + 5u_4 - 25v_1 - 348v_2 - 250v_3 - 2v_4 \leq 0 \quad (33)$$

$$224u_1 + 5u_2 + 5u_3 + 5u_4 - 23v_1 - 499v_2 - 200v_3 - 2v_4 \leq 0 \quad (34)$$

$$326u_1 + 5u_2 + 6u_3 + 5u_4 - 19v_1 - 420v_2 - 220v_3 - v_4 \leq 0 \quad (35)$$

$$273u_1 + 5u_2 + 5u_3 + 5u_4 - 17v_1 - 689v_2 - 200v_3 - 4v_4 \leq 0 \quad (36)$$

$$284u_1 + 5u_2 + 4u_3 + 5u_4 - 34v_1 - 822v_2 - 350v_3 - 5v_4 \leq 0 \quad (37)$$

$$204u_1 + 5u_2 + 4u_3 + 5u_4 - 10v_1 - 501v_2 - 100v_3 - 7v_4 \leq 0 \quad (38)$$

$$273u_1 + 5u_2 + 8u_3 + 5u_4 - 11v_1 - 719v_2 - 100v_3 - 4v_4 \leq 0 \quad (39)$$

$$183u_1 + 5u_2 + 4u_3 + 5u_4 - 11v_1 - 262v_2 - 100v_3 - 2v_4 \leq 0 \quad (40)$$

The formulation of the RSDEA can be solved using the LINDO software as shown Figure 1. The calculation is carried out 12 times, according to the number of DMUs to the case. Each DMU (this case the DMU is defined as major of department) has a different objective function with the same constraints.

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 0.8987476

VARIABLE	VALUE	REDUCED COST
U1	0.001473	0.000000
U2	0.000000	0.841860
U3	0.000000	0.841860
U4	0.000000	0.841860
U5	0.000000	17.825163
U6	0.000000	588.030579
U7	0.000000	200.570526
U8	0.000000	5.841860
V1	0.000000	0.000000
V2	0.001701	0.000000
V3	0.000000	0.000000
V4	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	0.998608
3)	0.113448	0.000000
4)	0.611443	0.000000
5)	0.456741	0.000000
6)	0.527462	0.000000
7)	0.568716	0.000000
8)	0.419433	0.000000
9)	0.496521	0.000000
10)	0.480314	0.000000
11)	0.598183	0.000000
12)	0.496521	0.000000
13)	0.251223	0.000000
14)	0.000000	0.000000
15)	0.101252	0.000000
16)	0.485109	0.000000
17)	0.386165	0.000000
18)	0.352211	0.000000
19)	0.220551	0.000000
20)	0.518608	0.000000
21)	0.233971	0.000000
22)	0.769542	0.000000
23)	0.979526	0.000000
24)	0.551476	0.000000
25)	0.820563	0.000000
26)	0.175954	0.000000

Figure 1. Efficiency value of DMU₁ using the LINDO software

This study proposed a new method called RSDEA to evaluate the effectiveness of performance to tackle uncertainty problems with deals data-driven indeed input and output. Assess the efficiency of the proposed model; a confirmed case of the college with 12 departments of majors, 4 inputs, and 4 outputs was studied, as shown in Table 1. The data inputs are mean values of lecturers, students, research of funding, employees. As well as the outputs data which is the mean values of alumni, research publications, and LPM. In this case, given value of $\alpha = 0.2$. Furthermore, the RSDEA develops by extended SDEA with the robust optimization method. In (15) is only to maximize the output without changing the inputs. For (16) until (27) are the constraints of the SDEA formula where all inputs change with outputs, then there are eight outputs.

(28) until (40) also the constraints from the DEA framework. The result of RSDEA shows in Table 2. The higher value of the optimal efficiency from Table 2 is 0,8987476 or represents 89% at DMU1. That means the optimal performance efficiency with the RSDEA method obtained 89% only at DMU1.

Table 2. Result of the efficiency formulation RSDEA

DMUs	Result of RSDEA formulation
DMU ₁	0.8987476
DMU ₂	0.7379925
DMU ₃	0.7274454
DMU ₄	0.7136675
DMU ₅	0.7692308
DMU ₆	0.7692308
DMU ₇	0.7692308
DMU ₈	0.6718858
DMU ₉	0.4292363
DMU ₁₀	0.7692308
DMU ₁₁	0.8316246
DMU ₁₂	0.7692308

4. CONCLUSION

We have presented perform the proposed RSDEA method to evaluate the efficiency with uncertainty multi-inputs and multi outputs. The development of RSDEA methods carries out using linear programming techniques. RSDEA methods have the optimal solution to evaluate the effectiveness of performance to tackle uncertainty problems. Recommendation for the implementation of further research considering extending this approach by examining the characteristics of data uncertainty at each input and output by providing the best scenario in constraints for maximizing or minimizing the objectives of the RSDEA.

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